Similarity and Congruence

Objectives

Students will know how to determine that two figures are similar or congruent by investigating figures that are similar and figures that are congruent. Then they will know how to prove that two figures are similar or congruent by using definitions, postulates, and theorems.

Core Learning Goals

2.1.1 The student will analyze the properties of geometric figures.

2.2.1 The student will identify and/or verify congruent and similar figures and/or apply equality or proportionality of their corresponding parts.

Materials Needed

Worksheets, calculator, paper, ruler, formula sheet, protractor, patty paper

Pre-requisite Concepts Needed

Students will need to be able to measure segments using a ruler, measure angles using a protractor, and be familiar with the theorems and postulates used to prove figures similar and congruent.

Approximate Time

One 30 minute lesson for the discovery lesson and up to 60 minutes for the practice, depending on how well students remember the above-mentioned pre-requisite skills.

Similarity and Congruence

Lesson Plan

Warm-Up/Opening Activity

Use the **Prediction Guide** to determine what students already know. Feel free to add/delete/change statements as they fit your class.

Accept students' answers and justifications and ask if the class thinks they are reasonable.

Answers:
1. Disagree
2. Disagree
3. Agree
4. Agree
5. Agree

Development of Ideas

Worksheet: **Comparing Sizes of Figures**

Ask students to work in groups of 2 or 3 to measure all of the segments listed using a ruler. It is helpful if all students measure using the same unit (suggest measuring in centimeters). It may be helpful to have the students change their ratios to a decimal (about 1.4) to show that the ratios are equal.

Answers: The ratio is about 9:6.5 [due to differences in printing and copying, you will need to check these measurements carefully]

1. 12 cm:8.6 cm (about 1.4)
2. 8.8 cm:6.1 cm (about 1.4)
3. 6.5 cm:4.5 cm (about 1.4)
4. 41.6 cm:29.4 cm (about 1.4)

2\. 90° and 90°

6. 85° and 85°
7. 102° and 102°
8. 105.6 cm^2:52.46 cm^2 (about 2)
9. The ratio of the lengths stays the same in an enlargement, the angle measures stay the same, the perimeters are in the same ratio as the enlargement, and the areas increase by $(\text{ratio})^2$.
10. The segment length in photo 1 should be about 7 cm.
11. The angle measure in photo 1 would be 20°.

Development of Ideas (Continued)

Explorations/Investigations

Present the concept and discover the SSS and SAS postulates of congruency and the SSS, SAS and AA postulates of similarity.

Activity One

Worksheet: **Congruent and Similar Triangle Investigation Activity One**

Organize students into groups of two. Each student needs to cut out the three strips of paper. Place the strips together corner-to-corner to create a triangle. Compare your triangle to your partner's and to other triangles in the class. Are they congruent?

Encourage the students to prove the triangles congruent using the definition of congruent triangles. Measure each side with a ruler and each angle with a protractor (measuring each side may not be necessary if students realize that everyone started with the same three strips of paper).

What was the minimum amount of information used to create these two congruent triangles? [three congruent sides- this demonstrates SSS theorem of congruency]

Can you replicate this process in the same manner? Explain. [Allow students to create another set of congruent triangles if they are not convinced. You can use the beginning of Activity Two to show this congruence.]

Answers: The triangles will all be congruent, which can be explained by the side-side-side triangle congruence theorem (some of the triangles may need to be flipped or rotated to show congruence). After the angles have been measured, ASA, SAS, or AAS congruency theorems can be used to prove the triangles congruent.

Activity Two

Worksheet: **Congruent and Similar Triangle Investigation Activity Two**

Only ONE person in each group: Using the three strips of paper from Activity One, fold and cut each strip of paper in half. Create a triangle using the one-half pieces of each of the original strips. Compare this triangle to the first one. Describe any similarities and differences. [the two triangles are not congruent, but are the same shape. These two triangles are similar. You may want to emphasize that all of the NEW triangles are congruent to each other- have students prove this to you using SSS] Help students trace the new triangle and label the sides the same as in Activity One.

Development of Ideas (Continued)

Activity Two (Continued)

Students will justify the similarity of the old and new triangles. Have students measure each side of the triangle and place the measure in the space requested on the worksheet. Note that the ratios of the lengths of each pair of corresponding sides are proportional.

Have students measure each pair of corresponding angles. Note that the measures of each pair of corresponding angles are congruent. The answers in #4 and #5 show that the triangles are similar.

Answers: The triangles will be similar to one another, with a ratio of 2:1 for all of the sides (original:new). In order to be similar, all of the sides of a triangle must be in the same ratio and all of the angles need to be congruent. The two triangles in this activity are similar.

Activity Three

Worksheet: **Congruent and Similar Triangle Investigation Activity Three**

Each student will copy the two sides and included angle using patty paper. Students should now draw segment AC to create a triangle. Lead a discussion about what parts of the triangle were given and how their measures compare with everyone else's [SAS].

Now, students will compare their triangle to the other triangles in the group. Have students justify that the triangles are congruent – or similar [by definition they are both. Encourage them to show that the corresponding three sides of the triangles are congruent and therefore the triangles are congruent.]

Ask students to locate the midpoint of $\overline{AB}$ and of $\overline{BC}$ and label these points D and E, respectively. Draw segment DE. Compare this triangle to triangle ABC. [the two are similar]. Use the definition of similar triangles to justify that the triangles are similar.

Lead a discussion about what parts of the triangle were given and how this is a minimal amount of information needed in order to prove two triangles similar [SAS].

Answers: The triangles were all created with the same lengths for two sides of the triangle and the same measure of the angle in between these sides. All of the triangles will be congruent because of the side-angle-side triangle congruence theorem. The two triangles, ΔABC and ΔDEB are not congruent because they have different side lengths. They are similar because the sides are proportional to one another and the angle measures are the same.

Similarity and Congruence

Development of Ideas (Continued)

Activity Four

Worksheet: **Congruent and Similar Triangle Investigation Activity Four**

Students will now trace two angles and use these to create two triangles. These directions may be difficult for students to follow- please try to demonstrate using the overhead and transparencies.

Have the students create two different triangles. Lead a discussion about why the two triangles are not congruent and that AA [or AAA] is not a way to prove two triangles congruent.

The two triangles they create will be similar. Give them time to measure the sides and show that the ratios of the corresponding sides are proportional. This will be a great revelation because the ratios of the corresponding sides will be different than the others they have seen in these activities (all of the former being 2:1). Also have the students show that the three angles are congruent to each other.

Lead a discussion about how the minimum amount of information needed to prove two triangles similar is that two corresponding angles must be congruent. [AA]

By the time they finish these investigations, they will be ready to believe that the ASA theorem of congruence works, or you can show this using a demonstrations. Discuss that ASA works for similarity as well and can really be the same as the AA theorem.

Answers: The two triangles, ΔTUN and ΔWAY, are not congruent because they have different side lengths. They are similar, however, by the angle-angle similarity theorem.

Worksheet: **Practice with Congruent and Similar Triangles**.

Answers: 1.

Statement	Reason
1. H is the midpoint of $\overline{QK}$	1. Given
2. $\overline{QH} \cong \overline{HK}$	2. Definition of midpoint
3. $\overline{QM} \cong \overline{KD}$	3. Given
4. $\overline{MH} \cong \overline{DH}$	4. Given
5. $\Delta QHM \cong \Delta KHD$	5. Side-side-side triangle cong.

Similarity and Congruence

Development of Ideas (Continued)

Answers to Practicing with Congruent and Similar Triangles (Continued)

2.

Statement	Reason
1. $\overline{AB} \perp \overline{ED}$	1. Given
2. $\angle DBA$ and $\angle EBA$ are right angles	2. Definition of perpendicular lines
3. $\angle DBA \cong \angle EBA$	3. All right angles are cong.
4. B is midpoint of $\overline{ED}$	4. Given
5. $\overline{DB} = \overline{BE}$	5. Definition of midpoint
6. $\overline{AB} = \overline{AB}$	6. Reflexive property
7. $\Delta ABD \cong \Delta ABE$	7. Side-angle-side triangle cong.

3.

Statement	Reason
1. R is midpoint of $\overline{PT}$, $\overline{QS}$	1. Given
2. $\overline{PR} \cong \overline{RT}$	2. Definition of midpoint
3. $\overline{QR} \cong \overline{RS}$	3. Definition of midpoint
4. $\angle PRQ \cong \angle SRT$	4. Vertical angles are congruent
5. $\Delta PRQ \cong \Delta TRS$	5. Side-angle-side triangle cong.

4.

Statement	Reason
1. ΔPQR & ΔTSR are right triangles	1. Given
2. $\angle PQR$ and $\angle TSR$ right angles	2. Definition of right triangles
3. $\angle PQR \cong \angle TSR$	3. All right angles are congruent
4. $\angle PRQ \cong \angle SRT$	4. Vertical angles are congruent
5. $\Delta PQR \sim \Delta TSR$	5.Angle-angle similarity theorem

5.

Statement	Reason
1. $DB = \frac{2}{3}AB$ $EB = \frac{2}{3}CB$	1. Given
2. $\angle B \cong \angle B$	2. Reflexive Property
3. $\Delta ABC \sim \Delta DBE$	3. Side-angle-side similarity theorem

Development of Ideas (Continued)

Worksheet: **Applications**

Answers:

1. The measurements shown can determine the distance because there is enough information to show that the triangles are congruent because of side-angle-side triangle congruence. The distance across the pond is 34 meters.
2. The triangles are congruent by angle-side-angle triangle congruence, so Ranger C is 11.3 km from the fire.
3. All of the triangles can be proved to be the same size by measuring all of the sides of each triangle (side-side-side triangle congruence).
4. The two triangles are similar by angle-angle triangle similarity.

   $$\frac{1.2}{7.5} = \frac{1.4}{AB} \text{ so that AB = 8.75 miles}$$

   The length of $\overline{AR}$, by the Pythagorean Theorem, is about 4.5 miles, so the distance between the towns through R is 4.5+7.5, or 12, miles. The cost of the projects would be \$42 million (12 miles x \$3.5 million per mile) around town or \$56.875 million (8.75 miles x \$6.5 million per mile) through town. It is cheaper to build the road around the town through point R.
5. $\frac{170}{310} = \frac{x}{840}$ so that x = 460.64 cm.

   The wall is approximately 461 cm tall.
6. The triangles are similar by of angle-angle similarity. So,

   $$\frac{x}{x+45} = \frac{70}{90} \text{ so that x = 157.5 miles}$$
7. $\frac{30}{72} = \frac{40}{x}$ so x = 96 meters
8. The triangles are similar by the angle-angle similarity theorem (using the given angles and that vertical angles are congruent). AB is $\frac{420}{840} = \frac{720}{x}$, or 1440 feet.

Closure

Go back to the Prediction Guide. Allow students to make corrections to their Prediction Guides since they have worked through this lesson. Discuss any corrections and the correct answers.

Supplemental/Follow-Up Materials

Please use the HSA prototypes from 2000, 2001, and 2002 to share problems like this with your students.

Similarity and Congruence

Prediction Guide

Directions: Read the statement. Place a check next to whether you Agree or Disagree with the statement. Justify your reason for choosing Agree or Disagree.

		Agree	Disagree
1.	If two figures are similar then they are congruent	______	______
2.	If the ratios of the length of corresponding sides of two triangles are equal, then the triangles are congruent.	______	______
3.	If triangles are similar then they have the same shape.	______	______
4.	If two triangles are congruent then each pair of corresponding angles are congruent.	______	______
5.	If two angles of one triangle are congruent to corresponding angles of another triangle, then the triangles are similar.	______	______

Similarity and Congruence

Comparing Sizes of Figures

Photo 1 is an enlargement of photo 2. Use a ruler to measure the length of $\overline{AB}$ in each photo. What is the ratio of the first length to the second length?

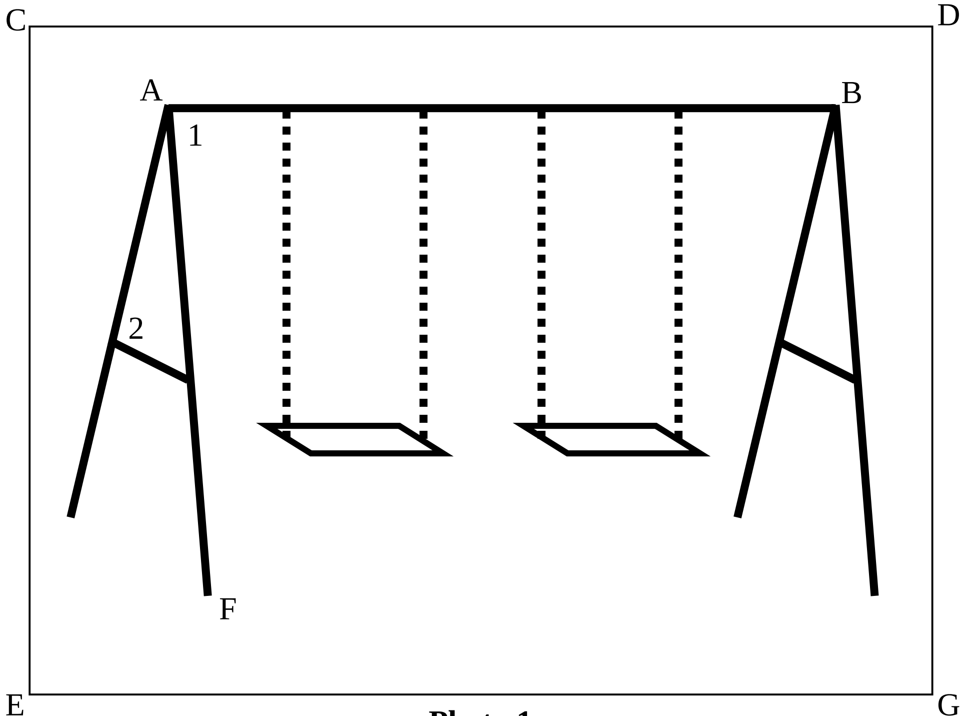


Photo 1

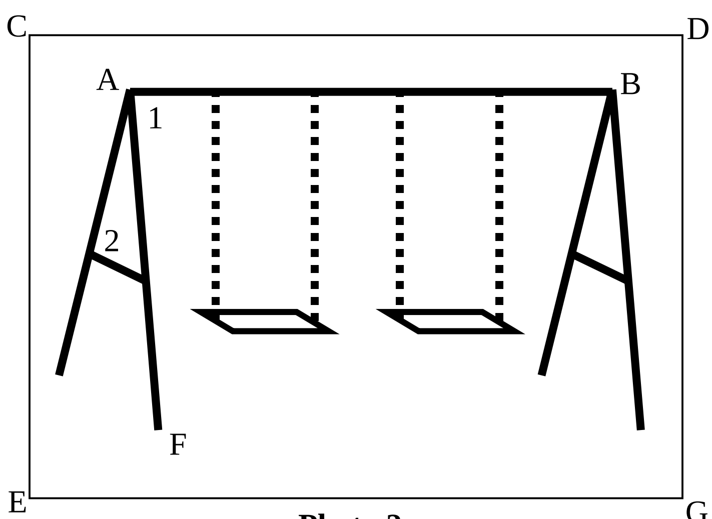


Photo 2

The ratio is ___________________________

Similarity and Congruence

Comparing Sizes of Figures (Continued)

In exercises 1-8, measure the quantity in photo 1 and photo 2 on the previous page. Find the ratio of the first measurement to the second measurement. You can measure lengths in centimeters or inches, but you must use the same units for each exercise.

1. $\overline{CD}$ = ______________

2. $\overline{CE}$ = ______________

3. $\overline{AF}$ = ______________

4. Perimeter of Quadrilateral CDGE = ________

5. $m\angle CDG$ = ________

6. $m\angle 1$ = ____________

7. $m\angle 2$ = ____________

8. Area of Quadrilateral CDGE = ____________

9. When a figure is enlarged, what appears to be true about corresponding lengths? Corresponding angles? Corresponding perimeters? Corresponding areas?

10. Suppose a segment in photo 2 has a measure of 5 centimeters. What would you expect the corresponding segment length to be in photo 1?

11. Suppose an angle in photo 2 has a measure of 20º. What would you expect the corresponding angle's measure to be in photo 1?

Similarity and Congruence

Congruent and Similar Triangle Investigation

Activity One

1. Cut out the three strips of paper below. Place the strips together corner-to-corner to create a triangle.

2. Compare your triangle to your partner's and to the other triangles in the class. What do you notice?

3. How can you prove that this is true?

4. Carefully place your triangle onto a piece of paper- place a piece of patty paper on top of your triangle and trace the inside of the triangle formed. Label Side 1 AB and Side 2 BC and then side 3 will be AC. Measure each angle.

5. Is your triangle congruent to your partner's triangle? Use mathematics to justify your answer.

Side 1 Side 1

Side 2 Side 2

Side 3 Side 3

Similarity and Congruence

Congruent and Similar Triangle Investigation

Activity Two

1. Only ONE person in each pair or group: Using the three strips of paper from Activity One, fold and cut each strip of paper in half. Create a triangle using one half of each of the original strips and place the strips together corner-to-corner.

2. Compare this triangle to the first one. Describe any similarities and differences.

3. Carefully trace this triangle and label the sides in the same way you did the first triangle in Activity One.

4. Place the smaller triangle inside the larger triangle and line up the corresponding parts. Measure each side of both triangles and write the ratios of the corresponding sides of the larger triangle to the smaller triangle below.

   Larger:
   Smaller:
   $\frac{AB}{AB} = \_\_\_\_$ $\frac{BC}{BC} = \_\_\_\_$ $\frac{AC}{AC} = \_\_\_\_$

   What do you notice about the ratio of the sides?

5. Measure each angle of both triangles.

   Larger: $\angle A =$ $\angle B =$ $\angle C =$

   Smaller: $\angle A =$ $\angle B =$ $\angle C =$

   What do you notice about the measures of the angles?

6. What things need to be true in order for two triangles to be similar? Are these two triangles similar? Use mathematics to justify your answer.

Similarity and Congruence

Congruent and Similar Triangle Investigation

Activity Three

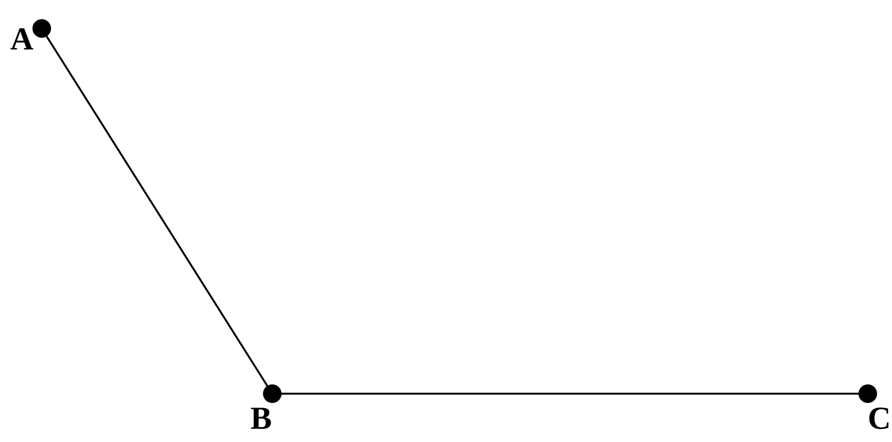
1. Using patty paper, copy the two segments and the included angle shown below. Draw segment AC using a straight edge.

2. What was the information that was given to you in order for you to create your triangle?

3. Compare your triangle to the other triangles in your group. What do you notice about your triangle compared to the others? Justify that this is true (can you use what you learned in Activity One?).

4. Locate the midpoint of $\overline{AB}$ and $\overline{BC}$ and label these points D and E, respectively. Draw segment DE. Is triangle ABC congruent to triangle DBE? Use mathematics to justify your answer.

5. Is triangle ABC similar to triangle DBE? Use mathematics to justify your answer.



Similarity and Congruence

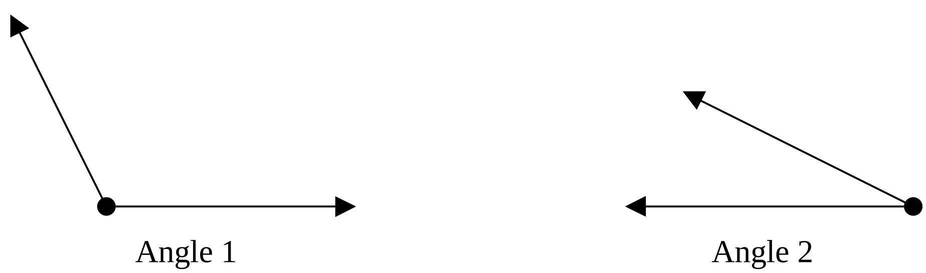
Congruent and Similar Triangle Investigation

Activity Four

1. Copy angle 1 and angle 2 shown below using two different pieces of patty paper. Place these two angles together in your own way to create part of a triangle. Using a third piece of patty paper, trace the angles and extend the sides to create a triangle. Name this ΔTUN.

2. Repeat this process and try to create a second triangle that looks different than ΔTUN. Name this ΔWAY. Is ΔTUN congruent to ΔWAY? Use mathematics to justify your answer.

3. Is ΔWAY similar to ΔTUN? Use mathematics to justify your answer.

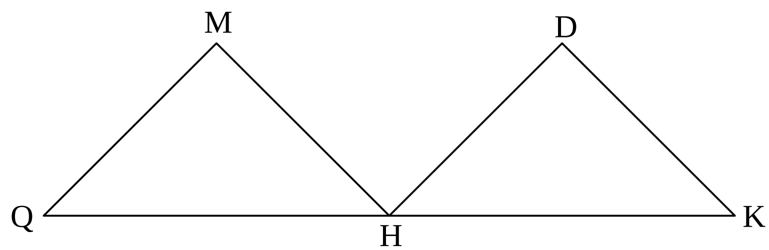


Similarity and Congruence

Practice with Congruent and Similar Triangles

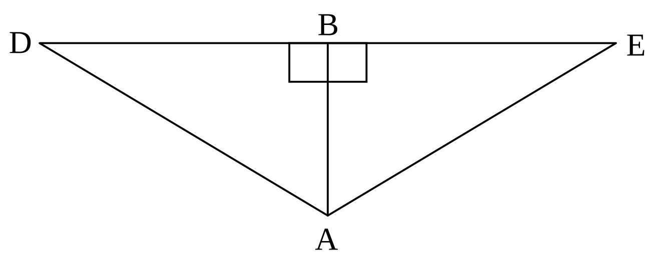
1. Given: H is the midpoint of $\overline{QK}$
   $\overline{QM} \cong \overline{KD}$
   $\overline{MH} \cong \overline{DH}$

   Prove: $\Delta QHM \cong \Delta KHD$



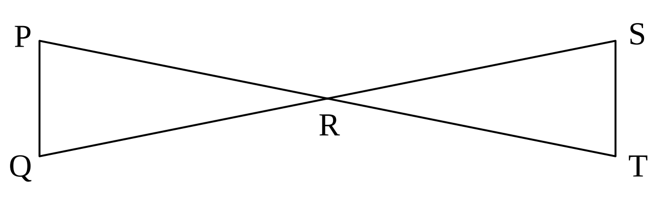
2. Given: $\overline{AB} \perp \overline{ED}$
   B is the midpoint of segment $\overline{ED}$

   Prove: $\Delta ABD \cong \Delta ABE$



3. Given: R is the midpoint of both $\overline{PT}$ and $\overline{QS}$

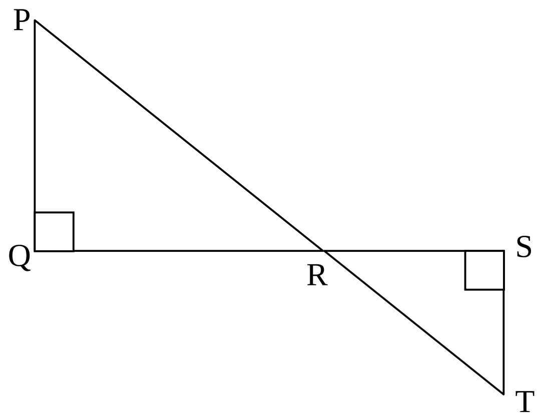
   Prove: $\Delta PRQ \cong \Delta TRS$



Practice with Congruent and Similar Triangles (Continued)

4. Given: ΔPQR and ΔTSR are right triangles

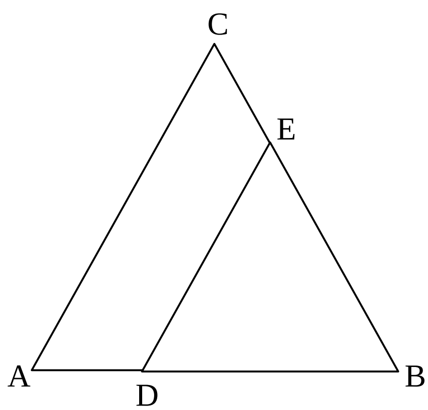
   Prove: $\Delta PQR \sim \Delta TSR$



5. Given: AB = CB = 9 cm

   DB = $\frac{2}{3}$ AB and EB = $\frac{2}{3}$ CB

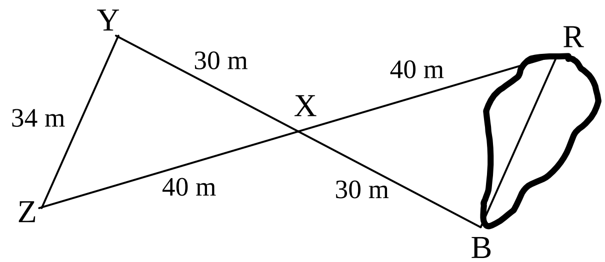
   Prove: $\Delta ABC \sim \Delta DBE$



Similarity and Congruence

Applications

1. A Forest Ranger needs to measure the distance across a pond from point R to point B. She knows the measures between the points shown below.

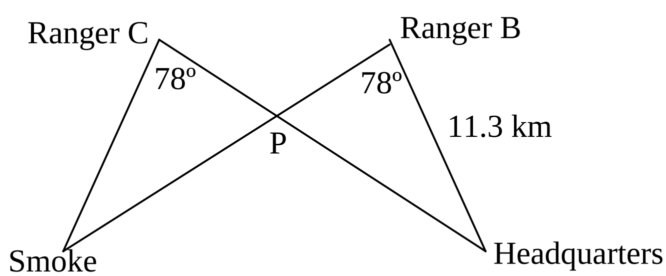


Note: The figure is not drawn to scale.

Describe how the measurements shown enable her to determine the distance across the pond.

What is the distance from R to B? Use mathematics to justify your answer.

2. Ranger B and her assistant, Ranger C, each sight a puff of smoke from two different ranger stations in the park. The smoke is located as shown, and point P is equidistant from the two rangers.



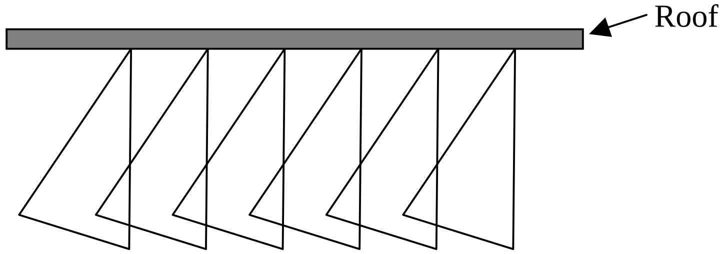
Note: The figure is not drawn to scale.

How close is Ranger C to the smoke? Use mathematics to justify your answer.

Similarity and Congruence

Applications (Continued)

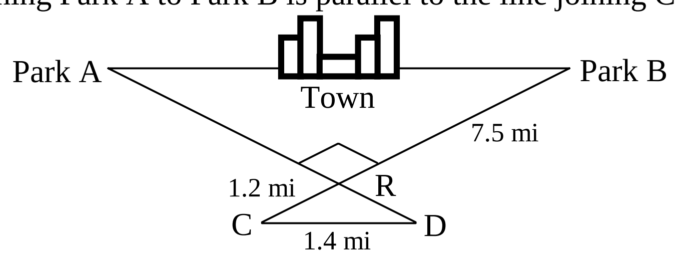
3. In the roof shown below, the triangles that support the roof must be congruent.



Note: The figure is not drawn to scale.

How can you prove to your boss that the triangles are congruent without measuring any angles?

4. Mason Construction wants to connect two parks on opposite sides of town with a road. Surveyors have laid out the map as shown. The road can be built through the town or around the town through point R. The roads intersect at a right angle at point R. The line joining Park A to Park B is parallel to the line joining C and D.



Note: The figure is not drawn to scale.

a. Determine the distance between the parks through the town. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

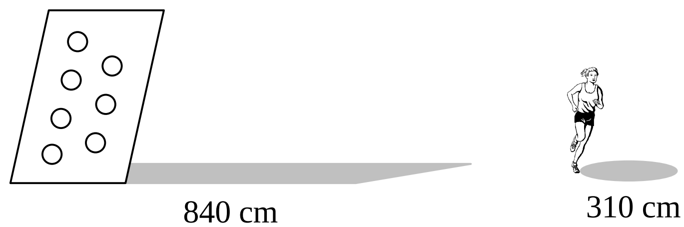
b. Determine the distance from Park A to Park B through Point R. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

c. If it costs \$3.5 million per mile to build the road around the town and \$6.5 million to build the road through the town, which road would be cheaper to build? Use mathematics to justify your answer.

Similarity and Congruence

Applications (Continued)

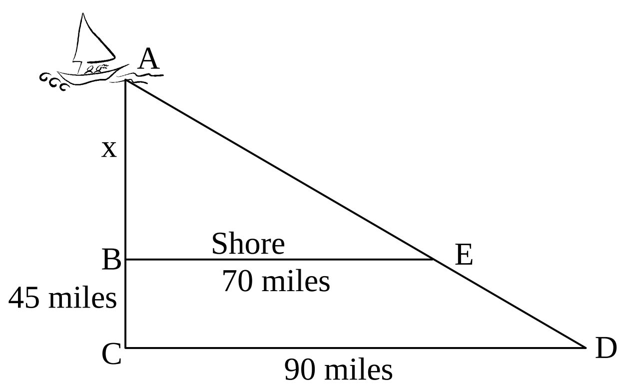
5. Marni wants to know the height of the climbing wall in the park. As she is standing next to the wall, Marni measures her shadow and the wall's shadow.



Note: The figure is not drawn to scale.

Determine the height of the wall if Marni is 170 cm tall.

6. Captain Cook needs to know the distance from his ship to the shore. He knows the measures given and that $\overline{BE} \parallel \overline{CD}$.



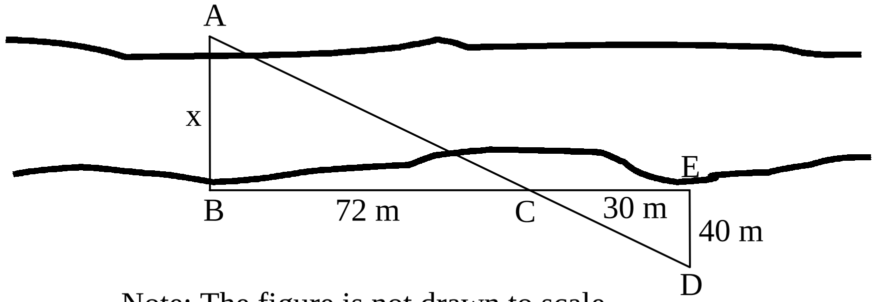
Note: The figure is not drawn to scale.

What is the distance (x) from his ship to the shore? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

Similarity and Congruence

Applications (Continued)

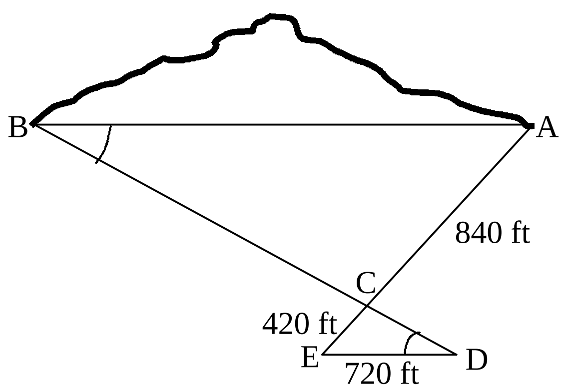
7. In the military it is often necessary to build temporary bridges across a river. To do this, it is necessary to determine the distance across the river. Sighting an object across the river, the engineer will set up right triangles to measure the distance across the river indirectly.



Note: The figure is not drawn to scale.

Using the diagram above, what is the length (x) of the bridge? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

8. A surveyor needs to determine the distance across the base (AB) of a mountain. This surveyor can directly measure the lengths given below.



Note: The figure is not drawn to scale.

a. Is ΔEDC similar to ΔABC? Use mathematics to justify your answer.

b. What is the measure of the base (AB) of the mountain?